A Three-dimensional, Implicit, Thermo-mechanical Computational Model for Polythermal Ice

A. Barone (Emory/SNL), M. Perego (SNL), A. Salinger (SNL)
M. Hoffman (LANL), G. Stadler (Courant), H. Zhu (UTA)

Introduction:

Temperate ice, a mixture of ice and melt water, is found near the bed of the Greenland and Antarctic ice sheets. Presently, only a few computational models for ice sheet dynamics can accurately model polythermal ice, i.e., model both temperate and cold (below melting point) ice. Most of these models are explicit, neglect the horizontal temperature diffusion and require temperature spin up of thousands of years for obtaining a steady state temperature field.

We present an **implicit**, fully three-dimensional computational model for the **steady-state** simulation of polythermal ice. The model is based on the enthalpy formulation proposed by Aschwanden et. al. (2012), with the addition of the gravity-driven moisture drainage model proposed by Hewitt and Schoof (2016). The enthalpy model is implicitly coupled with the First-Order (FO) ice sheet flow model. The resulting thermo-mechanical model is fully implicit and allows for the solution of the steady state for temperature and velocity without the need of performing a temperature spin up.

Mathematical Model:

It is convenient to model the temperature and porosity (water content) equations in terms of the enthalpy defined as

$$h = \rho c (T - T_0) + \rho_w L \phi$$

enthalpy temperature latent heat porosity (water content)

Relations between enthalpy, temperature and porosity are summarized in the table:

$$h_m := \rho c (T_m - T_0)$$

melting point enthalpy and temperature

	cold ice $h < h_m$	temperate ice $h \geq h_m$
T	$T = T_0 + \frac{1}{\rho c} h$	$T = T_m$
ϕ	0	$\frac{1}{\rho_w L}(h - h_m)$

Enthalpy equation reads:

$$\boxed{\mathbf{u} \cdot \nabla h + \nabla \cdot \mathbf{q} = \tau : \dot{\epsilon}}$$

ice velocity dissipation heat

$$\mathbf{q} = \begin{cases} -k \nabla T & \text{cold} \\ -k \nabla T_m + \rho_w L \mathbf{j} & \text{temperate} \end{cases} \qquad \mathbf{j} = -\nu \nabla \phi + \frac{1}{\eta_w} \left(k_0 \phi^\alpha (\rho_w - \rho) \mathbf{g} \right)$$

total flux water flux

At the bed interface we have the **Stefan condition**:

$$\rho_w L m = G + \tau_b \cdot \mathbf{u} - k \nabla T$$

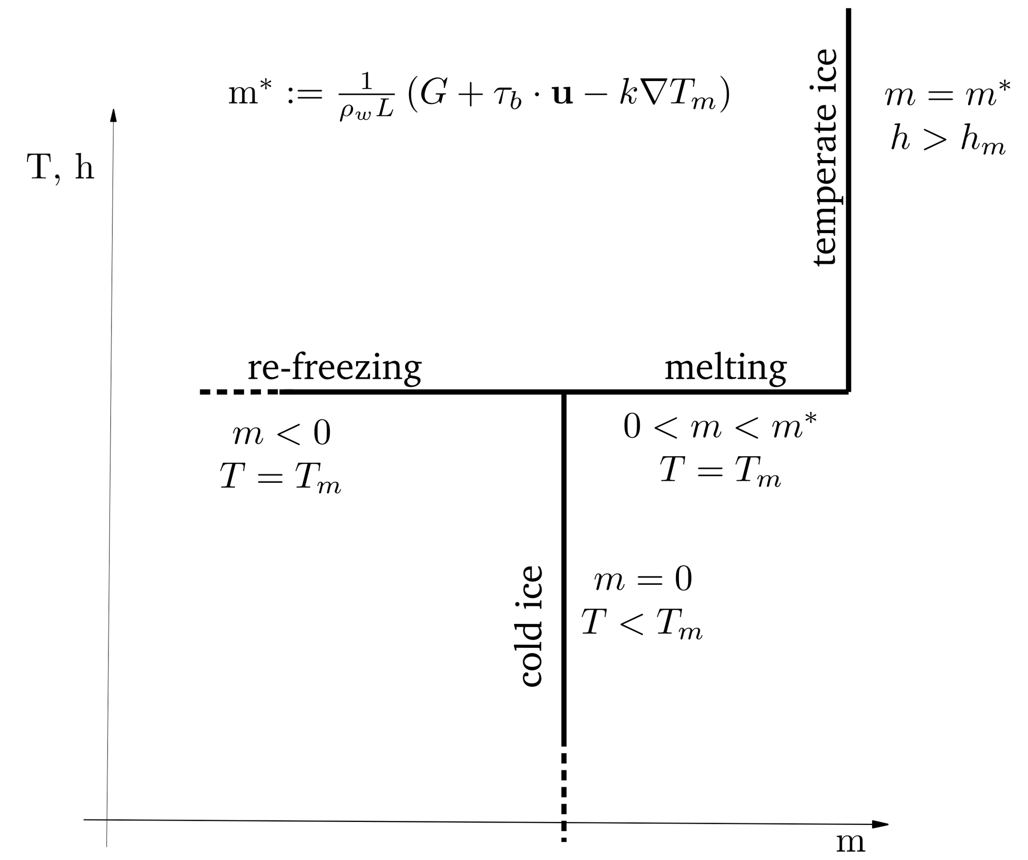
melting rate geothermal heat flux frictional heating

The enthalpy equation is coupled with an ice flow model. Temperature affects the ice viscosity and the vertical velocity b.c. through melting at the ice bed, and ice velocity is responsible for temperature advection and for the friction and dissipation heat terms. In our case, we couple the enthalpy equation with the FO equations.

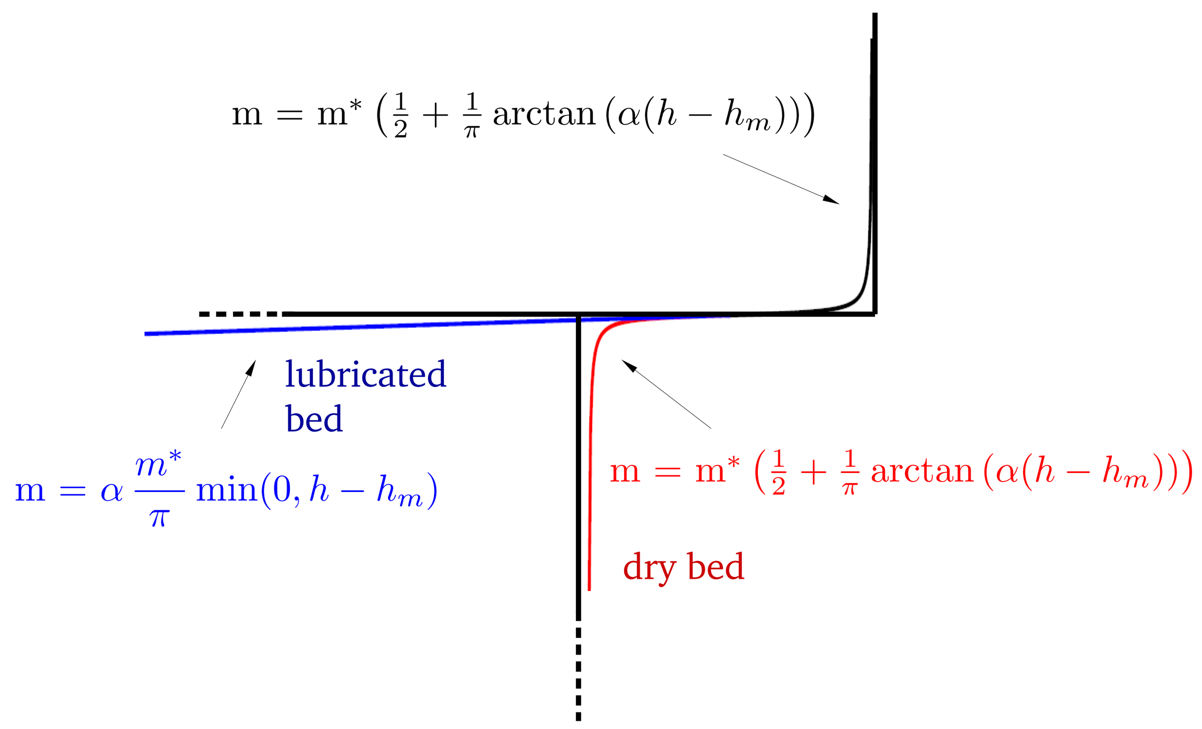
Numerical challenges:

The enthalpy model presents several strong nonlinearities because of the phase changes. In particular, at the bed interface, ice can be cold, melting, re-freezing or temperate. Modeling these different cases is challenging from a numerical point of view, especially for implicit large-scale solvers. Here we depict the relation between the melting rate and the temperature/enthalpy, in these different cases.

graph of (m, h) at the bed interface



We choose to approximate numerically the melting/enthalpy graph with the one shown below. Depending on whether the bed is lubricated or not, we follow the blue or the red curve. We perform a **parameter continuation** in order to get close to the original diagram.



α: continuation parameter (e.g. α goes form 1e-3 to 1e3)

Implementation:

Code implemented in parallel C++ finite element library **Albany**, within the **FELIX** module. **FELIX** implements FO ice flow model and features inverse/analysis capabilities for the estimation of poorly known fields such as basal friction and bed topography and for UQ. It is part of the DOE climate model (ACME) as a dycore of the land ice component of MPAS.

The enthalpy equation is advection-dominated, so we resort to the Streamline Upwind method for stabilization.

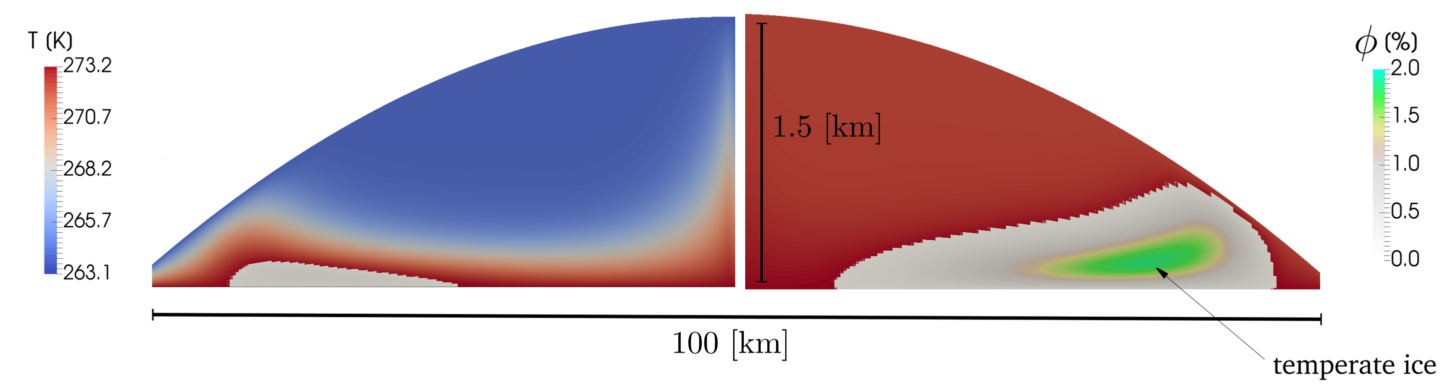
Albany heavily relies on Trilinos suites, in particular on **NOX** and **LOCA** for nonlinear solvers and algorithms for parameter continuations, which are essential for addressing the nonlinearities in the enthalpy model and on **BELOS**, **ML** and **Ifpack** for linear solvers and preconditioners.

Preliminary Results:

Dome problem: based on Hewitt and Schoof (in preparation).
We explore different scenarios and report, in each picture, the temperature (for cold ice) and porosity (for temperate ice).

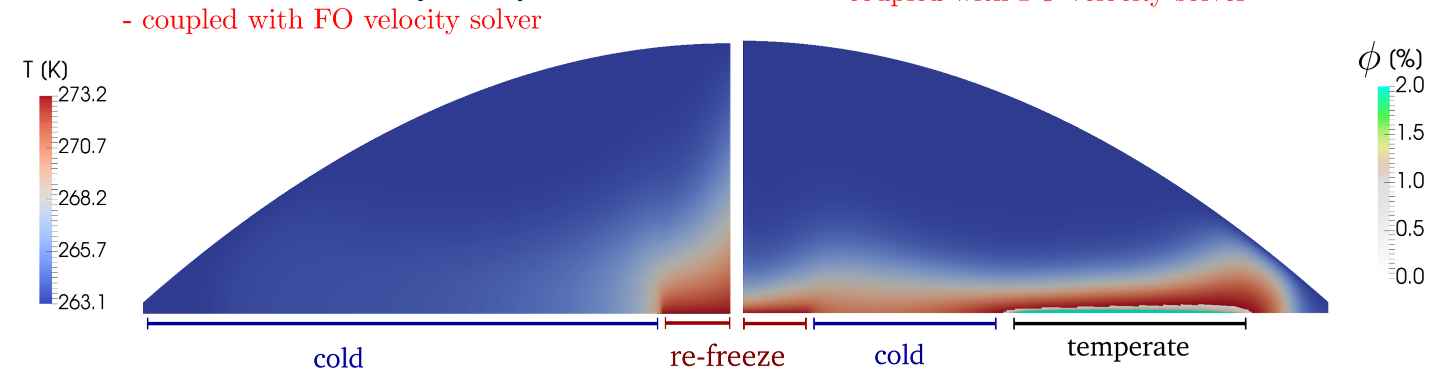
Problem 1: Settings
- top surface b.c.: T = -10 C
- bottom surface b.c.: h = h_m
- prescribed SIA velocity profile

Problem 2: Settings
- top surface b.c.: T = -1 C
- bottom surface b.c.: h = h_m
- prescribed SIA velocity profile



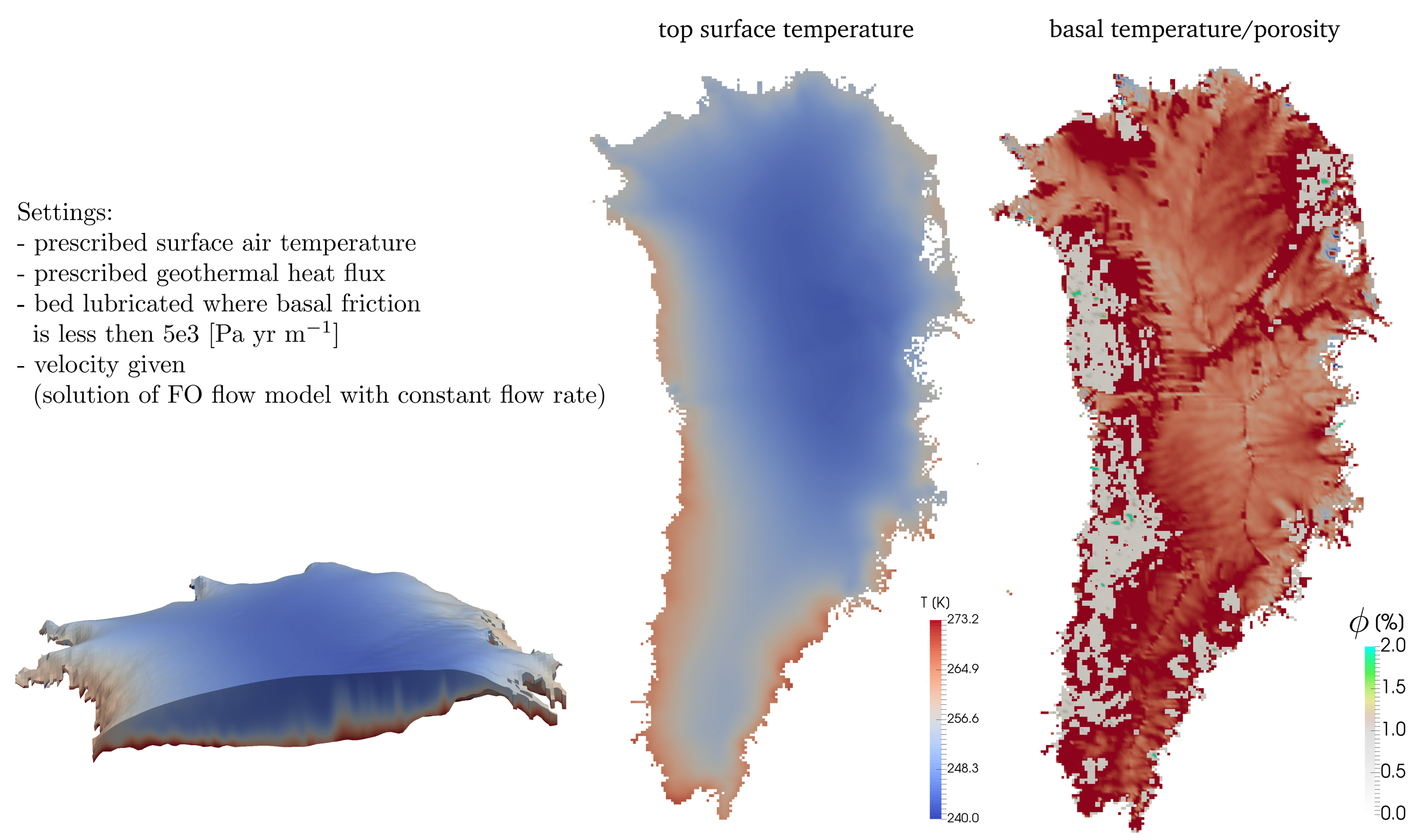
Problem 3: Settings
- top surface b.c.: T = -10 C
- no dissipation inside the dome
- bed lubricated near the center of the dome
- basal heat flux = 0.0 [W m^{-2}]
- coupled with FO velocity solver

Problem 4: Settings
- top surface b.c.: T = -10 C
- basal heat flux = 0.01 [W m^{-2}]
- bed lubricated near the center of the dome
- coupled with FO velocity solver



Towards Realistic Problems: 8km-resolution Greenland ice sheet

Settings:
- prescribed surface air temperature
- prescribed geothermal heat flux
- bed lubricated where basal friction is less then 5e3 [Pa yr m^{-1}]
- velocity given (solution of FO flow model with constant flow rate)



References:
- A. Aschwaden et al., *An Enthalpy formulation for glaciers and ice sheets*, Journal of Glaciology, 2012
- I. Hewitt, and C. Schoof: Models for polythermal ice sheets and glaciers, The Cryosphere Discuss., 2016
- C. Schoof and I. J. Hewitt, *A model for polythermal ice incorporating gravity-driven moisture transport*, Journal of Fluid Mechanics, 2016
- A. Barone and M. Perego, *Implementation of Enthalpy model for polythermal Glaciers*, CCR Summer Proceedings, 2016
- I. K. Tezaur et al., Albany/FELIX: *A parallel, scalable and robust, finite element, first-order Stokes approximation ice sheet solver built for advance analysis*, Geoscientific Model Development, 2015

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